1.6

10. a)
p is “I play hockey”
q is “I am sore”
r is “I use the whirlpool”

p--> q,
q--> r,
¬r
----
p-->r using hypothetical syllogism
¬q using modus tollens
¬p using modus tollens

c)
p(x) is “x is an insect”
q(x) is “x has six legs”
r(x) is “x is a dragonfly”
s(x) is “x is a spider”
t(x,y) is “x eats y”

∀x (p(x)->q(x))
∀x (r(x)->p(x))
∀x (s(x)->¬q(x))
∀x ∀y (s(x)∧r(y))→t(x,y)

∀x (r(x)->q(x)) using hypothetical syllogism
∀x (s(x)->¬p(x)) using contrapositive and then hypothetical syllogism

d)
q(x) is “x is a student”
p(x) is “x has an internet account”
r(x) is “x is homer”

∀x (q(x)->p(x))
∀x r(x)→¬p(x)

∀x(q(x)→¬r(x)) using contrapositive and then hypothetical syllogism

14. a) we can conclude this with modus ponens
b) we can conclude this using universal instantiation
d) we conclude this using existential instantiation and then existential generalization

16. a) this is a valid argument because it in order for one to have attended a university, they must have lived in a dormitory, and because the latter is false, the former must be false.
b) This is an invalid argument because although all convertibles are fun to drive, the converse is not necessarily true (all cars that are fun to drive are convertibles).
d) This is a valid argument because using universal instantiation, if for all xs, p(x) implies q(x), then p(a) implies q(a)

24. The errors are in steps 4 and 6. Universal generalization has the preconditions of P(a) for any arbitrary a.
28.

$\forall x\ (P(x) \vee Q(x))$
$\forall x((\neg P(x) \wedge Q(x) \rightarrow R(x))$

$\forall x\ (\neg P(x) \rightarrow R(x))$ using simplification
$\forall x\ (\neg R(x) \rightarrow P(x))$ using the contrapositive

1.7

6. Given a and be are odd and thus can be written as 2k+1 where k is some integer or 2m+1 where m is some integer:
(2k+1)(2m+1)
4km+2k+2m+1
2(2km+k+m)+1

Because we know that any integer multiplied by 2 is even, and one added to any even number is odd, we can conclude that the product of any two odd numbers is odd.

8. If n is a perfect square, than in can be written as s^2 where s is an integer. When we try to take the square root of s^2 + 2, we find that we can no longer pull an integer out of the square root. Thus, we can conclude that a perfect square plus 2 is not a perfect square.

16. We can first parities of m and n that do not result in an even product
case i) m is even, n is odd
case ii) m is even, n is even
case ii) m is odd, n is even (we do not pursue this case because we assume WLOG with case i)

We can also prove this by contraposition, if we show that if m and n are odd, then nm is odd. We did this above in question 6.
(2k+1)(2m+1)
4km+2k+2m+1

$2(2km+k+m)+1$

24. Using the pigeonhole principle, we see that as there are only 12 slots these days can fall into, and because we have 25 items to place in these slots, there exists no combination that does not satisfy the proposition that “at least three fall into at least on of the slots”.

30. p1: $a < b$
p2: $(a+b)/2 > a$
p3: $(a+b)/2 < b$

We can show that p3 -> p1:
$(a+b)/2 < b$
$a+b<2b$
$a<b$
We can show that p1-->p2:
$a<b$
$2a<b+a$
$a<(b+a)/2$
We can show that p2-->p3:
$(a+b)/2>a$
$a+b>2a$
$b>a$
$2b>a+b$
$b > (a+b)/2$

38. (assuming we consider 0 an integer) Statement: k = a^2 + b^2 + c^2, where k is a positive integer, and a,b,c are integers.

k=7 is a counter example because we can ignore all possible integers for a, b, or c such that their square is greater than 7, which leaves 0,1,2. With these possible values for a,b and c, we can see that we may achieve a maximum of 12 with all being equal to 2, and a minimum of zero when all being equal to 0. As we can see that two of these integers being 2 is too high (8) and only one being 2 and two being 1 is too low (6), we do not have enough fine-tune control to achieve a value of 7. I’d like to write a program that will, given the sum of the squares of any three integers, find all values in an interval for which this sum cannot take on.